

STEPS Math Quiz 3 Review

Wednesday, August 6, 2025

1. Differentiate each of the following functions (find $f'(x)$ or $\frac{dy}{dx}$ for each part)

(a) $f(x) = 3x^5 - 4x^3 + 2x^2 + 7x - 2$

(b) $f(x) = \frac{x^3 - 3x^2}{x}$

(c) $f(x) = \sqrt{x} + \frac{1}{x}$

(d) $f(x) = \frac{2x-1}{x^2+1}$

(e) $f(x) = \frac{\ln(x)}{x^2}$

(f) $f(x) = e^{-7x}$

(g) $f(x) = x^2 \cdot e^x$

(h) $f(x) = (2x^2 + 3)^6$

(i) $f(x) = \ln(3x^2 + 6x + 10)$

(j) $f(x) = \sqrt{5x + \frac{1}{x}}$

(k) $f(x) = e^{3x^2+2x}$

(l) $f(x) = (x^2 - 3)(x + 4)^3$

(m) $f(x) = 2^{4x^2-2}$

(n) $f(x) = \frac{1}{\sqrt{4x^2+2}}$

(o) $x^2y + y^2 = 7$

(p) $y^2 \ln(x) = x^2$

2. A balloon's radius is growing over time according to $r(t) = 3t^2 + 1$.

(a) Find the rate of change of the radius at time t .

(b) At what time is the radius increasing at a rate of 30 cm/s?

(c) If the volume of the balloon is $V = \frac{4}{3}\pi r^3$, express $\frac{dV}{dt}$ using the chain rule.

3. For a certain rectangle the length of one side is always three times the length of the other side.

(a) If the shorter side is decreasing at a rate of 2 inches/minute at what rate is the longer side decreasing?

(b) At what rate is the enclosed area decreasing when the shorter side is 6 inches long and is decreasing at a rate of 2 inches/minute?

4. A tank of water in the shape of a cone is being filled with water at a rate of $12 \text{ m}^3/\text{sec}$. The base radius of the tank is 26 meters and the height of the tank is 8 meters. At what rate is the depth of the water in the tank changing when the radius of the top of the water is 10 meters?

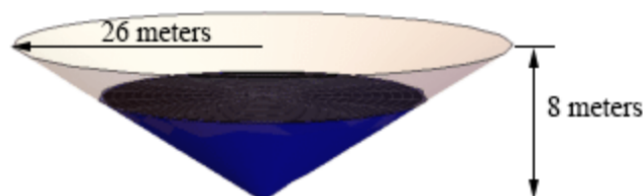


Figure 1: Water tank exercise

5. Find two positive numbers whose product is 810 and for which the sum of one and 10 times the other is a minimum.

6. We have 48 m^2 of material to build a box with a square base and no top. Determine the dimensions of the box that will maximize the enclosed volume.