STEPs Math Quiz 2

Thursday, July 24, 2025 Class time: 8:00 AM - 9:00 AM **Version B**

Name: ____________________ National ID: ____________________

Select your section instructor: ☐ Majid/Hamza ☐ Asaad

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1. Solve the following questions. Show your work.
   (a) Simplify the value of $e^{\ln(4)}$ = **4**
   (b) If $0 \leq x \leq \frac{\pi}{2}$ and $\sin(x)^2 - \cos(x)^2 = \frac{1}{2}$, find the value of $\tan(x)$ = $\sqrt{3}$

Solution:

$$\sin(x)^2 + \cos(x)^2 = 1$$

$$\Rightarrow 2\sin(x)^2 = \frac{3}{2} \Rightarrow \sin(x) = \frac{\sqrt{3}}{2}$$

$$2\cos(x)^2 = \frac{1}{2} \Rightarrow \cos(x) = \frac{1}{2}$$

$$\tan(x) = \frac{\sin(x)}{\cos(x)} = \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \sqrt{3}$$

2. For each of the following functions, determine if they are continuous at all real numbers. If not, state the points of discontinuity and its type (essential, removable, or jump). Show your work.

(a)

$$f(x) = \begin{cases} 2^x - 1 & \text{if } x \leq 0 \\ \log_{10}(x) & \text{if } x > 0 \end{cases}$$

Solution:

$$\lim_{x \to 0^-} f(x) = 2^0 - 1 = 0$$

$$\lim_{x \to 0^+} f(x) = \log_{10}(0) = -\infty$$

$\Rightarrow$ Discontinuous at $x = 0$ (essential)

(b)

$$g(x) = \begin{cases} \frac{x^2-4}{x-2} & \text{if } x \leq 0 \\ x^3 - 3x + 2 & \text{if } 0 < x \leq 2 \\ 2\sqrt{x+2} & \text{if } x > 2 \end{cases}$$

Solution:

$$\lim_{x \to 0^-} g(x) = \frac{0^2 - 4}{0 - 2} = 2$$

$$\lim_{x \to 0^+} g(x) = 0^3 - 3 \cdot 0 + 2 = 2$$

$\Rightarrow$ Continuous at $x = 0$

$$\lim_{x \to 2^-} g(x) = (2)^3 - 3 \cdot 2 + 2 = 4$$

$$\lim_{x \to 2^+} g(x) = 2\sqrt{2+2} = 4$$

$\Rightarrow$ Continuous at $x = 2$ $\Rightarrow$ Continuous at all real numbers

(c)

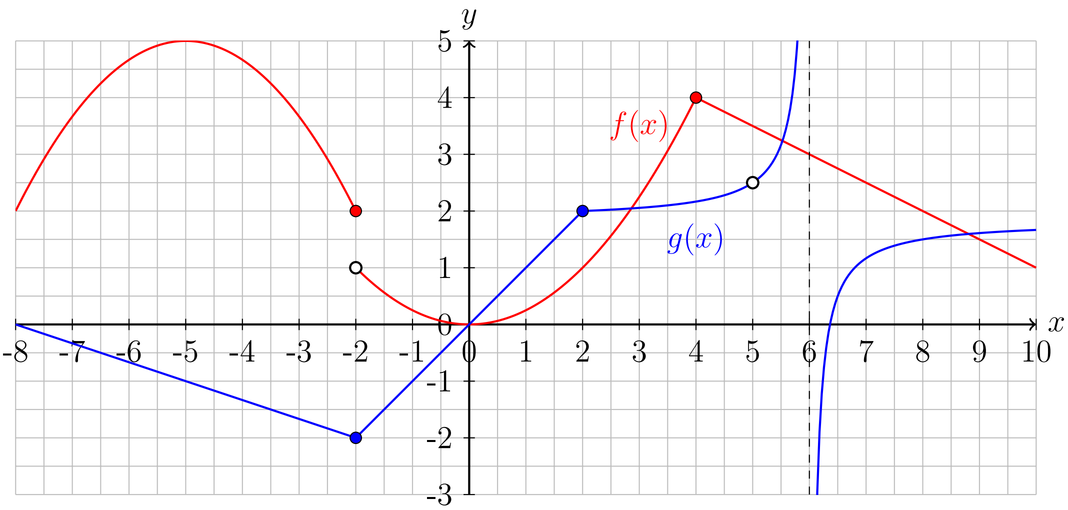
$$h(x) = \frac{x^2 + 7x + 12}{x^2 - 16}$$

Solution:

$$h(x) = \frac{(x+3)(x+4)}{(x-4)(x+4)}$$

$\Rightarrow$
Discontinuous at $x = 4$ (removable)
and Discontinuous at $x = -4$ (essential)

3. Below is the graph of functions $f(x)$ and $g(x)$. Determine the value of the following expressions.



(a)

$$\lim_{x\to 2}\frac{f(x)}{g(x)} = \boxed{2}$$

(b)

$$f(5) = \boxed{3.5}$$

(c)

$$\lim_{x\to -2^+} f(x) = \boxed{1}$$

(d)

$$\lim_{x\to -2^-} g(x) = \boxed{-2}$$

(e)

$$\lim_{x\to 6^+} g(x) = \boxed{-\infty}$$

(f)

$$\lim_{x\to 1}(g(x) - g(-x)) = \boxed{2}$$

4. Evaluate the following limits. Show your work.

(a)

$$\lim_{x\to 4}\frac{x-4}{2-\sqrt{x}} = \boxed{-4}$$

Solution: $\lim_{x\to 4}\frac{x-4}{2-\sqrt{x}} = \lim_{x\to 4}\frac{x-4}{2-\sqrt{x}}\cdot\frac{2+\sqrt{x}}{2+\sqrt{x}} = \lim_{x\to 4}\frac{(x-4)(2+\sqrt{x})}{4-x} = \lim_{x\to 4}\frac{(x-4)(2+\sqrt{x})}{-(x-4)} = \lim_{x\to 4} -(2+\sqrt{x}) = -4$

(b)

$$\lim_{x\to 2}\frac{\log_3(2x^2+1)}{x^2} = \boxed{\frac{1}{2}}$$

Solution: $\lim_{x\to 2}\frac{\log_3(2x^2+1)}{x^2} = \frac{\log_3(9)}{4} = \frac{2}{4} = \frac{1}{2}$

(c)

$$\lim_{x\to -2}(x^5 - 3\sqrt{-2x} + 1) = \boxed{-37}$$

Solution: $\lim_{x\to -2}(x^5 - 3\sqrt{-2x}+1) = (-2)^5 - 3\sqrt{-2(-2)} + 1 = -32 - 6 + 1 = -37$

(d)

$$\lim_{x\to\infty}\frac{4^{2x+1}+2^{4x}+1}{4^{2x}} = \boxed{5}$$

Solution: $\lim_{x\to\infty}\frac{4^{2x+1}+2^{4x}+1}{4^{2x}} = \lim_{x\to\infty} 4 + \frac{4^{2x}}{4^{2x}} + \frac{1}{4^{2x}} = 4 + 1 + 0 = 5$

(e)

$$\lim_{x\to 3}\frac{x^2-4x+3}{-x^2+2x+3} = \boxed{\frac{-1}{2}}$$

Solution: $\lim_{x\to 3}\frac{(x-3)(x-1)}{-(x-3)(x+1)} = \lim_{x\to 3}\frac{x-1}{-(x+1)} = \frac{3-1}{-(3+1)} = \frac{2}{-4} = \frac{-1}{2}$

(f)

$$\lim_{x\to 1}\frac{x^2-1}{x^3-1} = \boxed{\frac{2}{3}}$$

Solution: $\lim_{x\to 1}\frac{x^2-1}{x^3-1} = \lim_{x\to 1}\frac{(x-1)(x+1)}{(x-1)(x^2+x+1)} = \lim_{x\to 1}\frac{x+1}{x^2+x+1} = \frac{1+1}{1^2+1+1} = \frac{2}{3}$

(g)

$$\lim_{x\to 2^-}\frac{7}{x-2} = \boxed{-\infty}$$

Solution: $\lim_{x\to 2^-} \frac{7}{x-2} = \frac{7}{2^- - 2} = \frac{7}{0^-} = -\infty$

(h)

$$\lim_{x\to\infty} \frac{4^{2x}}{\sqrt{x+1}} = \boxed{\infty}$$

Solution: $\lim_{x\to\infty} \frac{4^{2x}}{\sqrt{x+1}} = \infty$ because 4^{2x} (exponential) grows faster than $\sqrt{x+1}$ (square root)

(i)

$$\lim_{x\to\infty} (\sqrt{x^2+2x} - x) = \boxed{1}$$

Solution: $\lim_{x\to\infty}(\sqrt{x^2+2x} - x) = \lim_{x\to\infty}(\sqrt{x^2+2x} - x) \cdot \frac{\sqrt{x^2+2x}+x}{\sqrt{x^2+2x}+x} = \lim_{x\to\infty} \frac{x^2+2x-x^2}{\sqrt{x^2+2x}+x} = \lim_{x\to\infty} \frac{2x}{\sqrt{x^2+2x}+x} = \lim_{x\to\infty} \frac{2}{\sqrt{1+\frac{2}{x}}+1} = \frac{2}{1+1} = 1$

STEPs Math Quiz 2

Thursday, July 24, 2025 Class time: 9:15 AM - 10:15 AM **Version B**

Name: ____________________ National ID: ____________________

Select your section instructor: ☐ Majid/Hamza ☐ Asaad

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1. Solve the following questions. Show your work.

   (a) Simplify the value of $e^{\ln(7)}$ = **7**

Solution: By properties of exponents and logarithms, we have

$$e^{\ln(7)} = 7.$$

   (b) If $\frac{\pi}{2} \leq x \leq \pi$ and $\sin^2(x) - \cos^2(x) = \frac{1}{3}$, find the value of $\tan(x)$ = $-\sqrt{2}$

Solution: We are given:

$$\sin^2(x) - \cos^2(x) = \frac{1}{3}.$$

Using the identity $\sin^2(x) + \cos^2(x) = 1$, let $\sin^2(x) = s$, so $\cos^2(x) = 1 - s$. Substituting:

$$s - (1 - s) = \frac{1}{3} \Rightarrow 2s - 1 = \frac{1}{3} \Rightarrow 2s = \frac{4}{3} \Rightarrow s = \frac{2}{3}.$$

So $\sin(x) = \sqrt{\frac{2}{3}}$, and $\cos(x) = \sqrt{\frac{1}{3}}$. But in $[\frac{\pi}{2}, \pi]$, $\sin(x) > 0$, $\cos(x) < 0$, so:

$$\tan(x) = \frac{\sin(x)}{\cos(x)} = \frac{\sqrt{\frac{2}{3}}}{-\sqrt{\frac{1}{3}}} = -\sqrt{2}.$$

2. For each of the following functions, determine if they are continuous at all real numbers. If not, state the points of discontinuity and its type (essential, removable, or jump). Show your work.

(a)

$$f(x) = \begin{cases} x^2 - x - 1 & \text{if } x \leq 2 \\ \log_2(x) & \text{if } x > 2 \end{cases}$$

(b)

$$g(x) = \begin{cases} \frac{x^2-1}{x+1} + 4 & \text{if } x \leq 0 \\ \sqrt{9 - x} & \text{if } 0 < x \leq 3 \\ \log_3(x) & \text{if } x > 3 \end{cases}$$

(c)

$$h(x) = \frac{x^2 + x - 2}{x^3 - 1}$$

Solution:

(a) Discontinuity at $x = 2$.
Left-hand limit: $2^2 - 2 - 1 = 1$, right-hand limit: $\log_2(2) = 1$.
Since both are equal and defined, **continuous at all real numbers**.

(b) Discontinuities at $x = -1$ and $x = 3$.
At $x = -1$: numerator $= 0$, denominator $= 0 \Rightarrow$ hole, **removable**.
At $x = 3$: $\sqrt{9 - x}$ is defined for $x \leq 3$, $\log_3(x)$ for $x > 0$. At $x = 3$,

$$\lim_{x \to 3^-} \sqrt{9 - 3} = \sqrt{6}, \quad \lim_{x \to 3^+} \log_3(3) = 1.$$

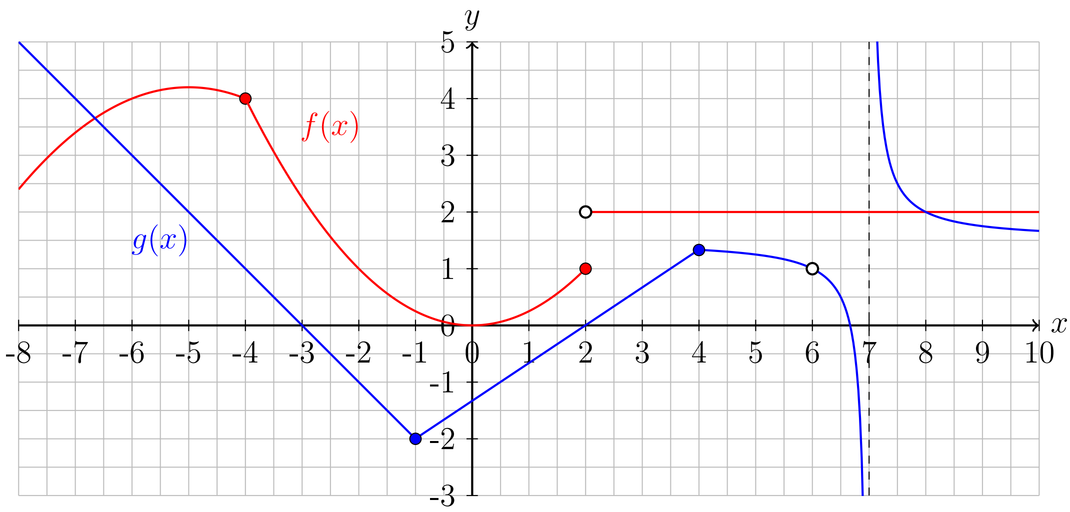
Not equal $\Rightarrow$ **jump discontinuity at** $x = 3$.

(c) Factor numerator and denominator:

$$h(x) = \frac{(x+2)(x-1)}{(x-1)(x^2+x+1)} = \frac{x+2}{x^2+x+1}, \quad x \neq 1.$$

So, removable discontinuity at $x = 1$.

3. Below is the graph of functions $f(x)$ and $g(x)$. Determine the value of the following expressions.



(a)

$$f(6) = \boxed{2}$$

(b)

$$\lim_{x\to 2} f(x) = \boxed{\text{Does not exist}}$$

(Left-hand limit from parabola: 1, right-hand limit from line: 2.)

(c)

$$\lim_{x\to 7^+} g(x) = \boxed{\infty}$$

(From graph: vertical asymptote at $x = 7$, right-hand limit $\to +\infty$)

(d)

$$\lim_{x\to -4} \frac{f(x)}{g(x)} = \frac{f(-4)}{g(-4)} = \frac{4}{1} = \boxed{4}$$

(From graph, $f(-4) = 4$, $g(-4) = 1$)

(e)

$$\lim_{x\to -2} f(x) + g(x)$$
$$= f(-2) + g(-2)$$
$$= 1 + (-0.5) = \boxed{0.5}$$

(f)

$$\lim_{x\to 7} g(-x)$$
$$= g(-7) = \boxed{4}$$

4. Evaluate the following limits. Show your work.

(a)

$$\lim_{x\to 9}\frac{3-\sqrt{x}}{9-x}=$$

Solution: Multiply numerator and denominator by the conjugate:

$$\begin{aligned}&=\lim_{x\to 9}\frac{3-\sqrt{x}}{9-x}\cdot\frac{3+\sqrt{x}}{3+\sqrt{x}}\\&=\lim_{x\to 9}\frac{9-x}{(9-x)(3+\sqrt{x})}\\&=\lim_{x\to 9}\frac{1}{3+\sqrt{x}}=\frac{1}{3+3}=\frac{1}{6}.\end{aligned}$$

(b)

$$\lim_{x\to 8}\frac{\sqrt[3]{x}-1}{x-1}=$$

Solution: Direct substitution:

$$\frac{\sqrt[3]{8}-1}{8-1}=\frac{2-1}{7}=\frac{1}{7}.$$

(c)

$$\lim_{x\to 4}(x^2-\sqrt{x}-\log_2(x^2))=$$

Solution: Substitute directly:

$$4^2-\sqrt{4}-\log_2(16)=16-2-4=\boxed{10}.$$

(d)

$$\lim_{x\to 0^-}9^{\frac{1}{x^2}}=$$

Solution: As $x\to 0^-$, $1/x^2\to+\infty$, so:

$$9^{1/x^2}\to\infty.$$

(e)

$$\lim_{x\to 2}\frac{2x^2-6x+4}{3x^2-3x-6}=$$

Solution: Factor:
Numerator: $2x^2-6x+4=2(x-1)(x-2)$
Denominator: $3x^2-3x-6=3(x-2)(x+1)$.
Cancel $(x-2)$, then:

$$\lim_{x\to 2}\frac{2(x-1)}{3(x+1)}=\frac{2(1)}{3(3)}=\frac{2}{9}.$$

(f)

$$\lim_{x\to 2}\frac{x^2-4}{x^2-5x+6}=$$

Solution: Factor:

$$\frac{(x-2)(x+2)}{(x-2)(x-3)}\Rightarrow\frac{x+2}{x-3}.$$

Then:

$$\lim_{x\to 2}\frac{x+2}{x-3}=\frac{4}{-1}=-4.$$

(g)

$$\lim_{x\to -3^+}\frac{-3}{(x+3)^2}=$$

Solution:
As $x\to-3^+$, $(x+3)^2\to 0^+$, so:

$$\frac{-3}{\text{small }+}\to-\infty.$$

(h)

$$\lim_{x\to\infty}\frac{2^{2x}}{3^x}=$$

Solution:

$$2^{2x}=4^x,\quad\text{so}\quad\lim_{x\to\infty}\frac{4^x}{3^x}=\left(\frac{4}{3}\right)^x\to\infty.$$

(i)

$$\lim_{x\to\infty}(\sqrt{4x^2+4x}-2x) =$$

Solution:
Multiply by conjugate:

$$= \frac{(4x^2+4x)-4x^2}{\sqrt{4x^2+4x}+2x}$$

$$= \frac{4x}{\sqrt{4x^2+4x}+2x}.$$

Divide numerator and denominator by x:

$$= \frac{4}{\sqrt{4+\frac{4}{x}}+2} \to \frac{4}{2+2} = 1.$$

STEPs Math Quiz 2

Thursday, July 24, 2025 Class time: 10:30 AM - 11:30 AM **Version B**

Name: ______________________ National ID: ______________________

Select your section instructor: ☐ Majid/Hamza ☐ Asaad

1. Solve the following questions. Show your work.
   (a) Simplify the value of $e^{-\ln(5)}$ $=$ ____________
   (b) If $\pi \leq x \leq \frac{3\pi}{2}$ and $\sin(x)^2 - \cos(x)^2 = \frac{1}{4}$, find the value of $\tan(x)$ $=$ ____________

2. For each of the following functions, determine if they are continuous at all real numbers. If not, state the points of discontinuity and its type (essential, removable, or jump). Show your work.

(a)

$$f(x) = \begin{cases} x^2 - 1 & \text{if } x \leq 1 \\ -\log_3(x) & \text{if } x > 1 \end{cases}$$

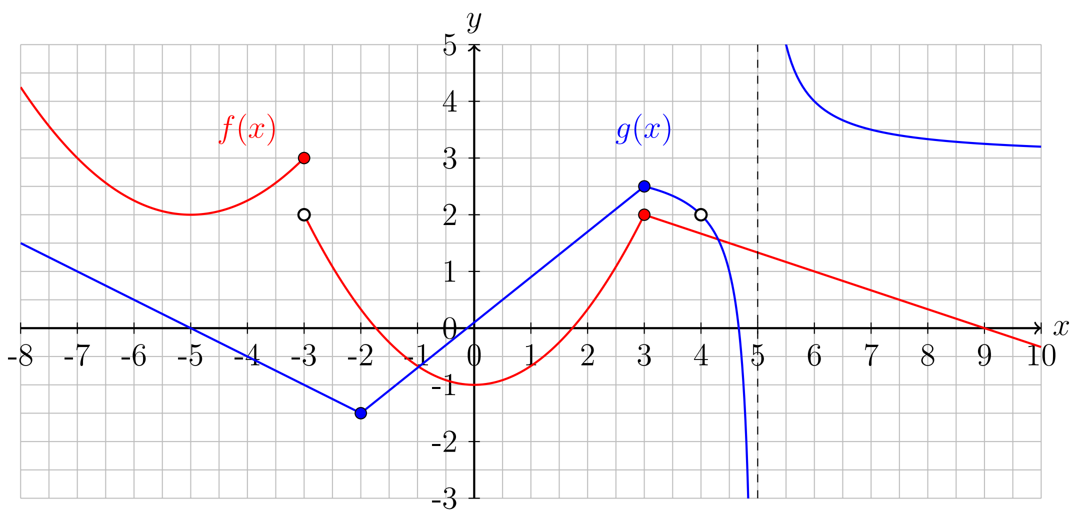
(b)

$$g(x) = \begin{cases} \frac{x^2-1}{x+1} & \text{if } x \leq 0 \\ -\cos(\pi x) & \text{if } 0 < x \leq 1 \\ \log_2(x+1) & \text{if } x > 1 \end{cases}$$

(c)

$$h(x) = \frac{x^2 - 1}{x^2 - 4x + 3}$$

3. Below is the graph of functions $f(x)$ and $g(x)$. Determine the value of the following expressions.



(a) $f(-3) =$

(b) $\lim_{x \to 3} f(x) =$

(c) $\lim_{x \to 4} g(x) =$

(d) $\lim_{x \to -7} \frac{f(x)}{g(x)} =$

(e) $\lim_{x \to 5^-} g(x) =$

(f) $\lim_{x \to 6} f(x) + g(-x) =$

4. Evaluate the following limits. Show your work.

(a)

$$\lim_{x\to 1} \frac{x-1}{\sqrt{x+3}-2} =$$

(b)

$$\lim_{x\to 5} \frac{x-3}{\log_4(3x+1)} =$$

(c)

$$\lim_{x\to -1} (2x^3 + \sqrt{-x} + 2^x) =$$

(d)

$$\lim_{x\to\infty} \frac{2^{3x+1} + 4^{x+1} + 1}{8^x} =$$

(e)

$$\lim_{x\to 0^-} 7^{\frac{1}{x^3}} =$$

(f)

$$\lim_{x\to 2} \frac{x^2-4}{x^2-5x+6} =$$

(g)

$$\lim_{x\to 2^-} \frac{-x}{\sqrt{x^2-4}} =$$

(h)

$$\lim_{x\to\infty} \frac{x^4 + 3x^3 + 2\sqrt{x}}{x^2 + 3^x} =$$

(i)

$$\lim_{x\to\infty} (x - \sqrt{x^2-4x}) =$$

(j)

$$\lim_{x\to 0^+} \log(1+x)^x =$$